

# Relationship between the Thickness of Stellar Disks and the Relative Mass of a Dark Galactic Halo

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**Abstract**—We analyze the  $R$ - and  $K_s$ -band photometric profiles for two independent samples of edge-on galaxies. The thickness of old stellar disks is shown to be related to the relative masses of the spherical and disk components of galaxies. The radial-to-vertical scale length ratio for galactic disks increases (the disks become thinner) with increasing total mass-to-light ratio of the galaxies, which reflects the relative contribution of the dark halo to the total mass, and with decreasing central deprojected disk brightness (density). Our results are in good agreement with numerical models of collisionless disks that evolved to a marginally stable equilibrium state. This suggests that, in most galaxies, the vertical stellar-velocity dispersion, on which the equilibrium-disk thickness depends, is close to a minimum value that ensures disk stability. The thinnest edge-on disks appear to be low-brightness galaxies in which the dark-halo mass far exceeds the stellar-disk mass. © 2002 MAIK “Nauka/Interperiodica”.

Key words: *galactic structure, galactic dynamics, edge-on galaxies*

## INTRODUCTION

Galaxy disks are complex structural components that include the bulk of the stellar mass in most of the spiral galaxies. Their masses and internal structure are crucial factors that determine all large-scale active processes in galaxies, such as propagation of density waves, star formation, and the associated phenomena.

As a rule, the brightness (and, consequently, the surface density) of disks at large galactocentric distances  $R$  decreases with increasing  $R$  according to an exponential law with a radial scale length  $h$  of the order of several kpc. Another geometrical parameter of the stellar disk—its thickness—can be characterized by vertical scale length  $z_0$ . In an isothermal disk, the decrease of density with the distance from the galactic plane can be described by the following law:

$$\rho(z) = \rho_0 \operatorname{sech}^2(z/z_0), \quad (1)$$

although some other alternative approximations are possible (de Grijs and van der Kruit 1996) such as the exponential or  $\operatorname{sech}(z)$ -model of brightness decrease.

The thickness  $z_0$ , or the vertical disk scale height, is primarily determined by the local disk density and stellar velocity dispersion. However, as we can see

in our own Galaxy, young and old stars have different velocity dispersions resulting in a rather complex vertical disk structure. Actually, since the bulk of the disk mass in spiral galaxies consists of stars that are several billion years old, hereafter we assign the photometrically determined thickness to the old stellar disk. Nonexistent or small color gradients in edge-on galaxies in the direction perpendicular to the disk plane beyond the narrow lane along the major axis where dust is concentrated (see de Grijs 1998 and references therein) are indicative of the rather homogeneous stellar content of old disks.

In contrast to radial scale length  $h$ , the disk thickness can be measured directly only in galaxies where disks are seen edge-on. The relative disk thickness can be characterized, to a first approximation, by the photometric isophotal axial ratios  $b/a$  of these galaxies, although inferring the vertical-to-horizontal scale length ratio  $z_0/h$  from photometric data requires modeling the 3D luminosity distribution of the disk (to make corrections for projection effect). The observed brightness distribution  $\mu(r, z)$  (determined by neglecting absorption) in a finite-thickness disk seen edge-on is related to the parameters  $h$  and  $z_0$  via modified Bessel's functions of the first kind,  $K_1(r/h)$  (van der Kruit and Searle 1981a):

$$\mu(r, z) = \mu(0, 0) \operatorname{sech}^2\left(\frac{z}{z_0}\right) \frac{r}{h} K_1\left(\frac{r}{h}\right), \quad (2)$$

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where  $r$  and  $z$  are the sky-plane coordinates.

Given relation (2),  $z_0$  and  $h$  can be determined from the vertical and major-axis photometric cross sections, respectively. At the peripheral regions of galaxies, such a parameter as the so-called disk cutoff radius,  $R_c$ , beyond which the decrease of the disk brightness is described by a shorter exponential scale length than at smaller galactocentric distances, may be of great importance. According to different estimates,  $R_c$  is typically equal to  $(3-5)h$  (see de Grijs *et al.* 2001; de Grijs and van der Kruit 1996; and references therein).

Indirect estimates of  $z_0$  may be based on measurements of the stellar velocity dispersion of the old disk population. However, such estimates require certain assumptions about the surface brightness or integrated mass of the exponential disk (Bottema 1993). The reverse is also true: given the disk thickness, velocity dispersion measurements make it possible to estimate the local surface brightness of the disk and, consequently, its total mass.

The observations of edge-on galaxies showed that the disk thickness varies over a wide range from one galaxy to another, and the apparent axial ratio can be as high as 10–20 for the thinnest disks (Kudrya *et al.* 1994; Karachentsev *et al.* 1997). What determines the relative disk thickness remains an open question. It appears to correlate with the morphological type, although the latter is determined rather uncertainly for edge-on galaxies—it is inferred not from the shape of spirals but only from the relative size and luminosity of the bulge. The disks of late-type galaxies (Sc–Sd) are, on the average, “thinner” than those of early-type objects (Karachentsev *et al.* 1997; de Grijs 1998; Ma *et al.* 1997, 1999). According to de Grijs (1998), the  $h/z_0$  ratio in his sample of edge-on galaxies varies from 1.5–2 for early-type spirals to 3–8 for Sc–Sd galaxies, and in addition, the relative disk thickness shows no direct correlation with the rotation velocity or luminosity. To illustrate these conclusions, in Fig. 1 we compare the ( $B$ -band)  $a/b$  ratio according to the Flat Galaxy Catalog (RFGC) (Karachentsev *et al.* 1999) with the known HI line width ( $W_{50}$ ), which is approximately equal to twice the maximum velocity  $V$  of disk rotation, and with absolute magnitude  $M_B$  (both parameters adopted from LEDA catalog).

It can be expected, however, from the most general considerations that the relative thickness of the equilibrium disk (at least its minimum possible value) must reflect its kinematic characteristics. The disk thickness is indeed determined by its surface density and by the dispersion  $C_z$  of stellar velocities in the direction perpendicular to the disk plane. On the other hand,  $C_z$  and radial dispersion  $C_r$  are interrelated

quantities<sup>1</sup> with the minimum  $C_r$  determined by the condition of local gravitational stability of the disk. Zasov *et al.* (1991) showed that if the radial dispersion  $C_r$  of stellar velocities in an old stellar disk is close or proportional to the critical threshold for the gravitational (Jeans) instability of the rotating disk and if velocity dispersion  $C_z$  along the  $z$ -coordinate is proportional to  $C_r$ , then the relative disk thickness should increase with decreasing relative mass of the galactic halo.

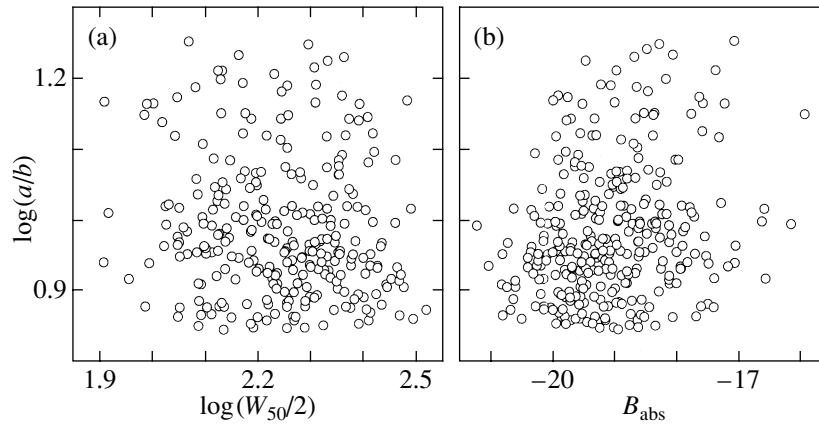
To a first approximation (neglecting the  $z$ -component of the acceleration due to the spherical component of the galaxy),  $z_0 \approx C_z^2/\pi G\sigma$  (here  $\sigma$  is the disk surface density). Let the radial velocity dispersion be equal to  $C_r = Q \times 3.36G\sigma/\kappa$ , where  $\kappa \sim V/R$  is the epicyclic frequency and the Toomre parameter  $Q = 1$  corresponds to a thin uniform disk that is stable (in Toomre’s sense) with respect to radial perturbations. In general,  $Q$  is a function of radius. Beyond the central bulge-dominated region, it varies slowly with  $R$  and gradually increases toward the periphery (Bottema 1993). Numerical models of marginally stable disks also show that parameter  $Q$  remains almost constant over a wide  $R$  interval beyond the central region and its value ( $Q \approx 1.2-1.5$ ) depends only slightly on the mass of the spherical and disk components of the galaxy (Khoperskov *et al.* 2002). Hence, for the region where one can assume  $Q(R) \approx \text{const}$ , the vertical-to-radial disk scale length ratio can be easily expressed in terms of other parameter ratios:

$$\frac{z_0}{h} \sim \frac{C_z^2}{\sigma h} \sim \frac{\sigma}{h\kappa^2} \sim \frac{\sigma h^2}{V^2 h} \sim \frac{M_d}{M_t}. \quad (3)$$

Here,  $M_d \sim \sigma h^2$  and  $M_t \sim V^2 h$  are the disk mass and the galaxy total mass, respectively, within the fixed radius (in the units of  $h$ ). The thinnest galaxies can therefore be expected to be those with the highest mass fraction of the spherical halo. This conclusion agrees well with the results of the 3D  $N$ -body numerical simulations of collisionless disks (Zasov and Morozov 1985; Zasov *et al.* 1991; Mikhailova *et al.* 2001).

When applied to real galaxies, the situation is further complicated by a number of factors that can potentially increase the thickness of quasi-equilibrium disks in the process of their long evolution and due to the slow increase of  $C_z$  (Gerssen *et al.* 2000;

<sup>1</sup>According to Gerssen *et al.* (2000), direct estimates obtained for several galaxies yield  $C_z/C_r \approx 0.5-0.8$ ; within approximately the same interval (0.35–0.8) fall the ratios obtained by numerical simulation of the dynamical evolution of initially “cold” collisionless disks (Mikhailova *et al.* 2001). The condition of stability against bending perturbations for collisionless disk yields  $C_z/C_r \approx 0.37$  (Polyachenko and Shukhman 1977).



**Fig. 1.** Diagrams illustrating the absence of correlation between the observed axial ratio  $a/b$  and the HI line halfwidth  $W_{50}/2$  (a) or absolute magnitude  $B_{\text{abs}}$  (b) for 340 galaxies from the RFGC catalog (Karachentsev *et al.* 1999).

Binney 2000). These factors include the scattering of disk stars during their interaction with giant molecular clouds or globular clusters; interaction of stars with density waves; merging of small satellites, which could repeatedly pass over the disk; star formation in the process of gas accretion onto the disk, which has not yet reached equilibrium; and gravitational perturbations due to neighboring galaxies. The latter effect shows up conspicuously in the fact that the relative thickness of disks in interacting systems is about twice greater than in galaxies without close neighbors (Reshetnikov and Combes 1997).

Note that the efficiency of all the processes mentioned above should be different at different galactocentric distances, whereas photometric measurements of edge-on galaxies imply that the disk thickness varies only slightly with radius (van der Kruit and Searle 1981a, 1981b; Barnaby and Thronson 1992). (However, some galaxies appear not to obey this rule—see de Grijs and Peletier 1997). The conclusion about the disk thickness remaining constant over a wide interval of galactocentric distances also follows from numerical  $N$ -body simulations of the dynamical evolution of initially cold (along the  $z$ -coordinate) collisionless disks (Mikhailova *et al.* 2001).

To clarify the processes that determine the vertical scale height of a stellar disk, it is worth verifying whether the relative thickness of disks seen edge-on correlates with the dark halo mass, and this is precisely the aim of this work.

### GALAXY SAMPLES

We chose the galaxies satisfying the condition  $a/b \geq 7$ , which is the underlying criterion of the Flat Galaxy Catalog (RFGC, Karachentsev *et al.* 1999). The objects obeying this criterion are mostly Sc–Sd galaxies ( $\sim 75\%$ ). These are disk-dominated galaxies

with a small bulge contribution to the integrated luminosity (although the bulge presence may be seen in central regions), making it easier to determine their vertical and radial scale lengths and the total disk luminosities.

In this work, we use two samples of edge-on galaxies.

The first sample consists of 121 late-type UGC galaxies and includes objects with  $a/b > 7$  (in the  $B$  band). For these galaxies,  $R$ -band surface CCD photometry was performed with the BTA telescope (Karachentsev *et al.* 1992). We excluded from the initial sample objects whose outer isophotes had an uncertain shape and those with isophotal asymmetry in the inner region, which might indicate that the disk inclination differs appreciably from  $90^\circ$ . We also excluded nearby galaxies ( $V < 750 \text{ km s}^{-1}$ ), Virgo members, and galaxies with large internal extinction ( $A_R > 0.5$ ). Our final analysis was based on a final sample of 51 galaxies.

Karachentsev *et al.* (1992) report their estimates of  $R$ -band axial ratios  $a/b$ , angular sizes of the semi-major axes of the 23 and  $24^m/\text{arcsec}^2$  isophotes, the corresponding isophotal magnitudes, and photometric profiles of the observed galaxies.

We estimated the radial scale length  $h$  by fitting the photometric major-axis profile to a function implied by relation (2) at  $z_0 = 0$ . Given  $h$ , the vertical scale height  $z_0$  can be determined by measuring the semi-major ( $a$ ) and semi-minor ( $b$ ) axes of a certain isophote of the galaxy (sufficiently far from the center to minimize bulge effects) and using relation (2). The latter implies the points lying along the major ( $0, a$ ) and minor ( $b, 0$ ) axes:

$$\text{sech}^2 \left( \frac{b}{z_0} \right) = \frac{a}{h} K_1 \left( \frac{a}{h} \right). \quad (4)$$

Unfortunately, the available photometric data were insufficient for allowing a more refined approach that makes use of the entire pattern of the two-dimensional brightness distribution of a galaxy. We found the galaxies of our sample to have the ratio of the mean semimajor axis to the radial disk scale,  $a/h$ , equal to 2.9 and 3.7 for the  $23^m$  and  $24^m/\text{arcsec}^2$ , respectively.

As a second sample, we analyzed the largest 60 RFGC galaxies, whose vertical ( $z_0$ ) and radial ( $h$ ) disk scale lengths could be determined in a more rigorous way—by modeling photometric cross sections along and across the major axis of the galaxy. To this end, we used the 2MASS survey  $K_s$ -band near-infrared images available from the NASA Extragalactic Database (NED). A detailed description of the procedure we used to determine the photometric parameters is given by Bizyaev and Mitronova (in press). The above authors obtained vertical disk scale heights averaged over 20–30 vertical cross sections. We fitted each profile to a  $I = I_0 \text{sech}^2(z/z_0)$  law with allowance for atmospheric blurring. We determined the radial disk scale lengths from the cross sections parallel to the major axis of the galaxy (but not closer than  $\sim 2''$  to avoid the dustiest regions). To minimize bulge effects in the estimated photometric disk parameters  $z_0$  and  $h$ , we also excluded the centermost regions in the cases where the isophotal ellipticity decreased centerward (due to the presence of the bulge). We treated  $z_0$  and  $h$  in equation (2) and the central brightness of the exponential disk as free parameters of the photometric model.

For a comparative analysis of disk scales, we selected the 24 brightest ( $K_s < 10^m 5$ ) and relatively distant ( $V > 750 \text{ km s}^{-1}$ ) galaxies of the second sample, excluding the objects with the strongest galactic extinction  $A_K > 0.25$  and probable Virgo cluster members.

The first sample of galaxies with  $R$ -band photometry and the initial sample of galaxies from the 2MASS catalog have 28 objects in common. When comparing the two samples, we excluded two objects with supposedly nonexponential profiles yielding highly discrepant scale length estimates obtained in two samples (UGC 542 = RFGC 206 and UGC 7774 = RFGC 2336). In Fig. 2, we compare the independently determined  $z_0$  and  $h$  (in arcseconds). The median radial and vertical scale height ratios for the two samples are equal to  $h(\text{BTA})/h(2\text{MASS}) = 1.21 \pm 0.08$  and  $z_0(\text{BTA})/z_0(2\text{MASS}) = 1.66 \pm 0.07$ , respectively. The relation between the radial scale lengths agrees well with the conclusion of de Grijs (1998) that near-infrared ( $K$ ) photometric scale lengths are systematically smaller than those measured at shorter wavelengths (by a factor of about

$\approx 1.2$  and  $\approx 1.6$  compared to the  $I$ - and  $B$ -band data, respectively). The scale length ratio  $z_0/h$  also decreases as one passes to longer wavelengths (see Fig. 6 of de Grijs (1998)). According to our measurements, the relative thickness of the galaxies of the first sample ( $R$ -band) is also greater than that of the second sample ( $K_s$ -band) (Fig. 3): the means ( $h/z_0$ ) are equal to  $3.52 \pm 0.1$  and  $4.93 \pm 0.34$  for BTA and 2MASS samples, respectively. This effect, however, can be partially due to systematically overestimated  $z_0$  based on BTA data, because the method employed is sensitive to the eventual bulge effects in the computed minor axes of the isophotes used to determine the vertical scale height.

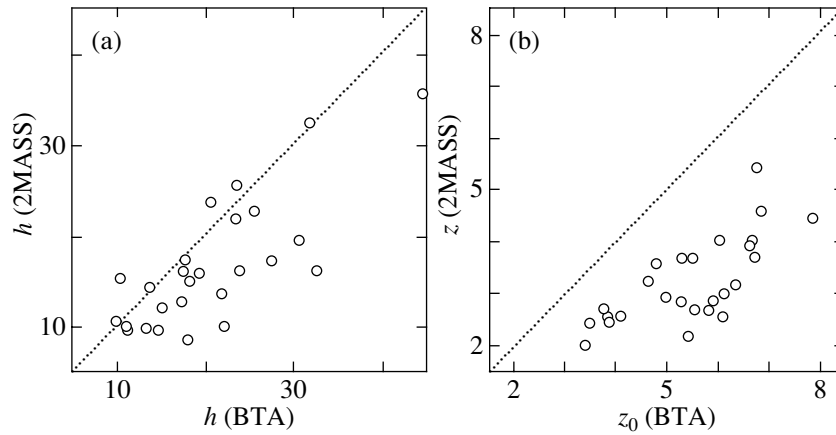
Our photometry showed that the galaxies of the first sample (BTA data) have a mean integrated color index of  $B-R = 1.06 \pm 0.05$ . The mean color indices of the edge-on galaxies common for the two samples are  $B-K_s = 3.34 \pm 0.17$  and  $R-K_s = 2.24 \pm 0.12$ , respectively. These results agree well with the integrated colors of Sc–Sd galaxies of 86 galaxies observed almost face-on (de Jong 1996). The absence of strong reddening is no surprise: the dust lane extending along the major axis of an edge-on galaxy can strongly decrease the observed luminosity while having little effect on the color if the optical depth of the galaxy  $\tau \gg 1$ .

#### VERIFICATION OF THE DEPENDENCE OF THE RELATIVE DISK THICKNESS ON A $M/L$ RATIO OF A GALAXY

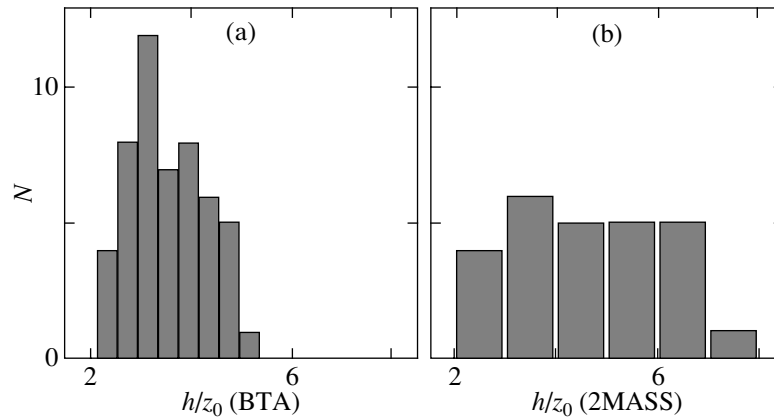
If the above assumption about the decrease of the relative disk thickness with the mass fraction of the spherical component (dark halo) is true,<sup>2</sup> one should expect the  $z_0/h$  ratio to be the lowest in galaxies with a high ratio of the integrated mass to the integrated red (infrared) luminosity: the latter is only slightly sensitive to ongoing star formation and therefore better corresponds to the total mass of the stellar population of the disk.

Hereafter, we determine the masses of galaxies inside a fixed radius of  $R_m = 4h$ , within which the luminosity (actually, the disk luminosity) was determined from photometric data. Less than 10% of the mass of the exponential disk is located beyond  $4h$ —even in the absence of the usually observed steepening of the radial distribution at large  $R$ . We assume that the total mass  $M_t$  of the galaxy within  $R_m$  is approximately

<sup>2</sup>The samples considered consist mostly of late-type galaxies without massive bulges, and, therefore, the bulk of the mass of the spherical component belongs to the dark halo.



**Fig. 2.** Relations between the estimated radial (a) and vertical (b) disk scale lengths in different color bands (BTA— $R$  band; 2MASS— $K_s$  band) for the galaxies common for both samples.



**Fig. 3.** Histogram of the radial-to-vertical disk scale length ratio,  $h/z_0$ , for the galaxy samples considered: (a)  $R$ -band (BTA data); (b)  $K_s$ -band (2MASS data).

equal to  $W_H^2 R_m / 4G$ , where  $W_H$  is the HI line width.<sup>3</sup> This simple expression for mass is, strictly speaking, correct for spherically-symmetric systems, however, this assumption introduces a rather small error. Numerical simulations of galaxies with the measured velocity dispersion of the old disk stellar population imply halo masses significantly exceeding the disk masses within chosen  $R_m$  in most of the cases (Zasov *et al.* 2000; Khoperskov *et al.* 2001). However, even if the mass of the halo is equal to that of the thick disk within  $R_m = 4h$ , the above formula overestimates the mass  $M_t$  only by  $\approx 25\%$ .

In the first sample, we estimated the galaxy luminosities from the  $24^m/\text{arcsec}^2$  isophotal magnitudes

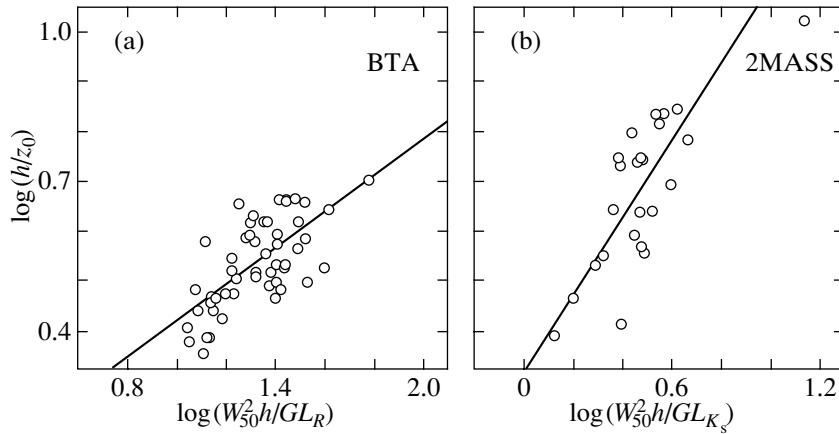
<sup>3</sup>In this work, we use the  $W_{50}$  width at 50% of the maximum (adopted from LEDA database). However, the choice between  $W_{20}$  and  $W_{50}$  is of no fundamental importance, because both quantities are close to twice the maximum velocity of gas rotation.

extrapolating them if necessary out to  $R_m$  based on the radial brightness scale length.

If the relative component masses are unknown, the mass of the disk cannot be estimated from the rotation velocity and therefore we infer it from the disk luminosity assuming that  $M_d = A(\lambda)L_d \times (M/L)_d$ , where  $A(\lambda) > 1$  is the factor that allows for internal extinction (which is important for the  $R$  band);  $L_d$ , the observed disk luminosity, and  $(M/L)_d$ , the integrated mass-to-luminosity ratio of the stellar population in the chosen photometric band. The total-to-disk mass ratio can therefore be written in the following form:

$$\frac{M_t}{M_d} \approx \frac{W_{50}^2 h}{A(\lambda)GL_d(M/L)_d}. \quad (5)$$

The luminosity underestimation of an edge-on galaxy is difficult to take into account: it can be important even in the infrared. The extinction correction applied to reduce the  $R$ -band magnitudes of edge-on galaxies to those of face-on galaxies exceeds, on



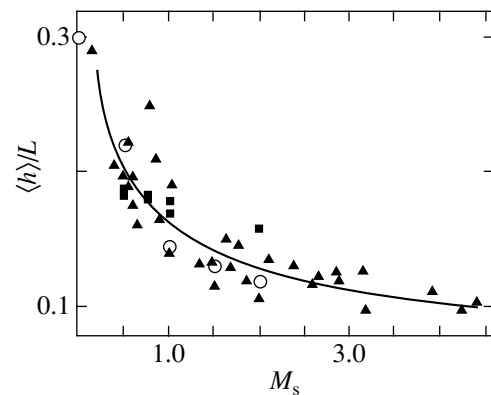
**Fig. 4.** Relation between the photometrically determined radial and vertical disk scale length ratio and the quantity  $W_{50}^2 h / GL_{(R, K_s)}$ , which determines the  $M_t / L_d$  ratio within  $R = 4h$ : (a)  $R$  band (BTA data); (b)  $K_s$  band (2MASS data).

the average,  $1^m$  (Tully *et al.* 1998). The problem of estimating  $M_d$  from the observed luminosity is further complicated by the large scatter of coefficient  $A(\lambda)$ , which, in turn, can depend on the disk mass and thickness. For the second galaxy sample, the photometric estimates should be much less affected by dust. The reasons for this are twofold: (1)  $K_s$ -band extinction in galaxies resulting from their edge-on orientation does not, on the average, exceed  $0^m.3$  (Tully *et al.* 1998), and (2) when estimating the scale lengths we excluded the regions close to the Galactic plane, which suffer from the strongest extinction. However, in spite of the simplifying assumptions adopted here both the first and the second galaxy samples exhibit conspicuous relations between  $h/z_0$  and  $M_t/L_d$  (or, to be more precise, a quantity proportional to this ratio)—see Fig. 4—with the correlation coefficients equal to 0.68 and 0.73, respectively. This relation, which corroborates the conclusion that the disk thickness decreases with the relative mass of the spherical component is the main result of this work.

The scatter of data points on the diagrams shown here is due to the errors in the estimates of the parameters used the differences of  $A_\lambda$  and  $M/L_d$  of the stellar populations of individual galaxies, and the disregarded physical factors, which may increase the disk thickness (see the Introduction). The differences between the slopes based on two galaxy samples must be real despite the uncertainty of the inferred slope of the relation in Fig. 4a (photometric estimates based on 2MASS data are more reliable): the shallower behavior of the  $R$ -band relation agrees qualitatively with the fact that thinner galaxies (in the upper part of the diagram) are more extinction-affected and thus have their  $M/L$  overestimated.

## DISCUSSION AND CONCLUSIONS

This study is probably the first to show that a relation exists between the relative disk thickness and the masses of the spherical and disk components of real galaxies. The conclusion that marginally stable collisionless disks become thinner with increasing mass fraction of their spherical components (in the absence of external gravitational perturbations) was earlier reached from  $N$ -body numerical simulations of three-dimensional disks in a fixed field of the spherical component starting from an unstable state with low vertical velocity dispersion (Zasov *et al.* 1991; Mikhailova *et al.* 2001). During the time equal to several rotation periods, at the outer disk edge the parameter  $C_z$  increases reaching a certain level (which decreases with  $R$ ) as a result of the development of



**Fig. 5.** Relation between the rms distance  $\langle h \rangle$  of points from the disk plane expressed in the units of the radial disk scale length and the halo-to-disk mass ratio,  $M_s$ , obtained from  $N$ -body numerical simulations of galaxies with marginally stable disks. The figure is adopted from Mikhailova *et al.* (2001, Fig. 2). In our notation  $\langle h \rangle / L$  is  $z_0 / h$ .

bending perturbations that increase stellar velocity dispersion along the  $z$ -coordinate. Eventually, the disk becomes marginally stable against both perturbations in its plane and the bending perturbations. See Khoperskov *et al.* (2001, 2002) for a detailed description of numerical simulations.

Figure 5 (same as Fig. 2 in the paper by Mikhailova *et al.* (2001)) compares the relative disk thickness and  $M_s = (M_t - M_d)/M_d$ —the spherical-to-disk mass ratio—based on the results obtained by constructing numerical models for galaxies with different component masses and different shapes of rotation curves corresponding to those actually observed  $V(R)$  in real galaxies.

To compare the relations shown in Figs. 4 and 5, one must convert disk luminosities into disk masses. Assuming, like we did in the previous section, that  $W_{50}^2 h/G$  determines the total mass of the galaxy within  $R_m = 4h$ , we can write the quantity  $M_s$  laid off along the horizontal axis in Fig. 5 as

$$M_s = \frac{W_{50}^2 h}{G \times M_d} - 1. \quad (6)$$

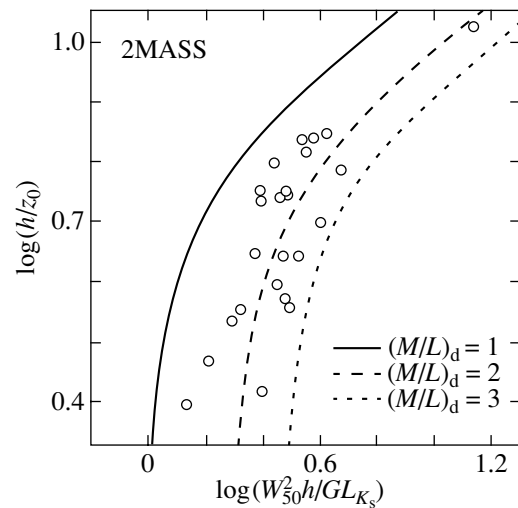
It follows from this equation that

$$\frac{W_{50}^2 h}{G \times L_d} = (M_s + 1)(M/L)_d, \quad (7)$$

where  $(M/L)_d$  is the disk mass-to-luminosity ratio for the chosen spectral interval.

Figure 6 shows in a logarithmic scale the diagram given in Fig. 4 with the superimposed curve from Fig. 5 computed in accordance with equation (7) for three mass-to- $K_s$ -band luminosity ratios ( $M/L_d = 1, 2$ , and 3).

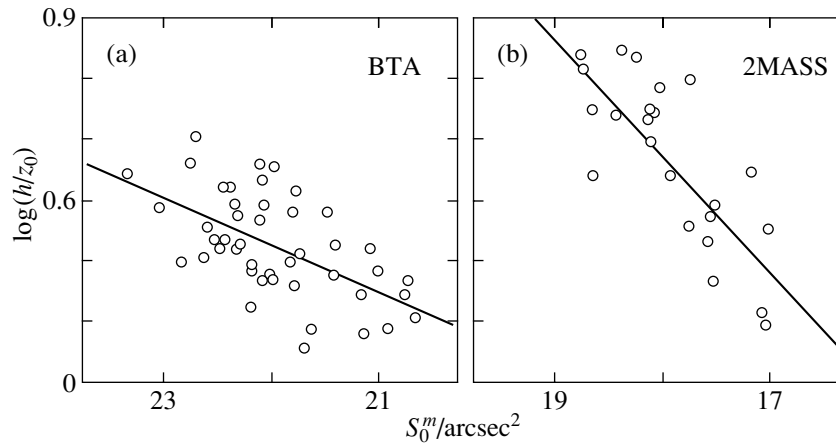
Evolutionary models yield for the stellar population of cosmological-age galaxies a mass-to-luminosity ratio of  $(M/L)_{\text{model}} \approx 1$  for the photometric  $K$  band, which is close to  $K_s$  (Bell and de Jong 2001). This ratio remains somewhat uncertain due to the lack of data about the low-mass end of the stellar mass function. All galaxies are actually situated in the domain between the adopted ratios, which are quite reasonable for a stellar population. This shows that models of marginally stable disks agree well with observations. This leads us to conclude that for most of the galaxies the mechanisms of additional disk heating (scattering by massive clouds, tidal perturbation of the disk) are not crucial for the formation of the vertical structure. The approximately constant disk thickness along the radius is then a result of two opposite tendencies—the radial decrease of both the surface disk density and the velocity dispersion at which the disk reaches stable equilibrium—almost exactly cancelling each other.



**Fig. 6.** The same diagram as in Fig. 4b with superimposed collisionless-disk relation inferred from numerical simulations (the line in Fig. 5) recalculated with three disk mass-to-luminosity ratios (in solar units):  $(M/L)_d = 1, 2$ , and 3.

Figure 7 compares the relative disk thickness with the face-on central surface brightness  $S_0$  (in magnitudes) reduced to the face-on position using model  $R$ - and  $K_s$ -band brightness distributions. The correlation between these parameters is even more conspicuous than that between  $z_0/h$  and  $(M/L)_t$ , although the existence of such a relation is nothing unexpected: indeed, “normal” and low surface brightness galaxies were already shown to exhibit a close relation between  $S_0$  and integrated  $(M/L)_t$ , which characterizes the dark halo mass fraction (MacGaugh and de Block 1998). The lower the central surface brightness (and, consequently, the surface density), the higher the dark halo mass fraction within the chosen  $R = R_m$ . The correlation between these two quantities implies, in particular, the existence of a single linear (in the logarithmic terms) Tully–Fisher relation (luminosity or mass of the disk-rotation velocity) for galaxies with different surface brightness  $S_0$  (MacGaugh and de Block 1998).

This relation manifests itself most conspicuously in the diagram shown by Bizyaev and Mitronova (2002), which is based on an analysis of a 153-galaxy sample from the 2MASS survey. This relation appears to be more scattered at longer wavelength bands (as is evident from a comparison of diagrams a and b in Fig. 7); Bizyaev and Kaisin (in preparation) and Bizyaev (2000) came to the same conclusion based on  $R$ - and  $I$ -band photometry, respectively. The corresponding dependence in the  $B$  band is the least conspicuous (see Fig. 9 in the paper of de Grijs (1998)). The differences between the correlation coefficients and the slopes of the relations shown in Fig. 7 are



**Fig. 7.** Radial-to-vertical disk scale length ratio as a function of deprojected central disk brightness in magnitudes in the (a)  $R$  and (b)  $K_s$  band.

evidently due to selective internal extinction, which is strongest in galaxies with thin disks and becomes more important at shorter wavelengths. Underestimated brightness (or overestimated  $S_0$ , which is expressed in magnitudes) for galaxies with “thin” disks results in an underestimated slope of the relation in Fig. 7. The relation is more conspicuous in the  $K_s$  band where the flux decrease due to internal extinction amounts only to several tenths of a magnitude.

Thus the relations obtained lead us to conclude that the thinnest edge-on galaxies must be (after deprojection to a face-on orientation) low surface brightness spirals whose observed brightness is enhanced by the projection effect and whose dark-halo mass significantly exceeds the mass of the disk.

Note, however, that not all disk galaxies appear to obey the “dark-halo mass–disk thickness” relation. Relatively thick disks are observed not only in interacting systems (Reshetnikov and Combes, 1997) but also in some Irr galaxies, for which some authors pointed out a deficit of systems with strong apparent flattening (Hodge and Hitchcock 1966; Thuan and Seitzer 1979; van den Bergh 1988), and all this in spite of the fact that the dark-halo-mass fraction in low luminosity galaxies is, on the average, higher than in galaxies with high luminosity (Persik and Salucci 1996; Ashman 1992; Cote *et al.* 2000). Relatively thick disks observed in Irr galaxies may be a result of, among other things, a certain threshold level of stellar velocity dispersion, which cannot be lower than the velocity dispersion of gaseous clouds (usually  $\approx 10 \text{ km s}^{-1}$ ). Unfortunately, the strong contribution of young stars to the disk luminosity in Irr galaxies and their nonuniform distribution within the galaxy complicate the vertical photometric disk structure and increase the uncertainty of the photometric tilt estimates in comparison to what we have in the case

of spiral galaxies, thus preventing any direct comparison of the stellar disk thickness of these two types of objects.

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